Development of Boundary Condition Independent Reduced-Order Models using POD

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ABSTRACT

An introduction to boundary condition independent reduced-order modeling of complex electronic components using the Proper Orthogonal Decomposition (POD)-Galerkin approach is presented in this extended abstract. Current work focuses on how the POD methodology can be used with the Finite Volume Method (FVM) to generate reduced-order models that are boundary condition independent. The proposed methodology is demonstrated for the transient 1D heat equation, and preliminary results are presented.

INTRODUCTION

Boundary condition independent Compact Thermal Models, introduced by the DELHPI (DEvelopment of Libraries of PHysical models for an Integrated design environment) project [1], [2] has created a revolution in the field of model development for thermal analysis of electronic components. Alternate approaches [3][4][5] have been proposed to develop models for complex geometries with multiple heat sources. Application of Proper Orthogonal Decomposition has been suggested for the purpose of model reduction of complex electronic systems by Shapiro[6]. In their work[6][7], they follow a reduce-and-interconnect approach, where reduced-order for components are generated independently and then connected using a frequency response approach. Further application of POD-Galerkin approach in the area of electronics cooling can be found from the doctoral dissertation of Rambo[8]. A review of the literature shows few papers published in the application of POD to Finite Volume Methods. Majority of the work [9][10] involves the Finite Element Methods as it is commonly used for heat conduction analysis, because of the easy access to the mass and stiffness matrices. At present, the popular software packages that are being used in the electronics cooling industry are based on finite volume solvers. Therefore, it is important to identify methods that can be easily implemented in the existing system through small modifications or by the addition of third party subroutines. In this regard, the current research is built over the application of POD method as proposed by Astrid [11].

POD is a statistical method of looking at the characteristics of a particular data set. The first step is to obtain a complete data set that is relevant. In the proposed work, the relevant data set will be formed from data collected under different boundary conditions. By using POD, a low-order subspace that captures the maximum characteristics of the data is identified. The full dynamics of the governing equations in the discrete form is then projected on to this subspace. Projection could be achieved by different methods such as Least Squares, Collocation and Galerkin. In the present study, Galerkin projection is employed. This projection then yields a reduced-order model. Although the concept of POD has been around for about a century, it has been only in recent times POD has been recognized as a viable methodology for reducing complex systems for computational purposes. A detailed introduction to the mathematical basis of POD is available in the doctoral dissertations of Astrid [11], Rambo [8] and Atwell [12].

The "Method of Snapshots" introduced by Sirovich [13] is used to construct the original data set. For example, in the time-dependent 1D heat equation case, the snapshot could be the set of spatial temperature profile through the length of the rod over different time steps. The matrix formed by this method is termed as the "snapshot matrix". Singular Value Decomposition (SVD) or Eigenvalue Decomposition is carried out on the snapshot matrix to identify the eigenvalues and eigenvectors. The eigenvalues are then arranged in a descending order. A plot of the eigenvalues is known as the eigenvalue spectrum. The eigenvalue spectrum shows that the maximum energy is stored by the first few eigenvalues. A POD basis is constructed by arranging the eigenvectors corresponding to those eigenvalues that store the maximum energy. These eigenvectors are orthonormal to each other, and they capture the major characteristics of the entire data. This constitutes the POD subspace that represents the reduced order model. By Galerkin projecting the equations in the discreet form onto the POD subspace, solving for variables of the governing equation in the complete discretized form, has been converted to solving for the POD coefficients. The numbers of POD coefficient are in the order of the chosen POD basis. Thus, this results in a major reduction of the degree of freedom.

FORMATION OF POD BASES

The POD basis in finite dimensional Hilbert space can be obtained from the data in the following method. Let the snapshot matrix be defined as the solution matrix that contains temperatures at different spatial and temporal positions of the 1D transient heat equation.

$$T_{snap} = \begin{matrix} [T(1, time\_initial) \ldots\ldots\ldots\ldots T(1, No\_time\_steps)] \\ [\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots] \\ [\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots] \\ [\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots] \\ [T(No\_grid\_points, time\_initial) \ldots\ldots T(No\_grid\_points, No\_time\_steps)] \end{matrix} \quad (1)$$

The covariance matrix C is now defined as $C = \frac{1}{N} T_{snap} T_{snap}^{T}$ (2)

Singular value decomposition or Eigenvalue decomposition is done on C.

Eigenvalue decomposition: $C\phi_i = \lambda_i \phi_i$ (3)

Singular value decomposition: $C = U\,\mathrm{E}\,V^*$ (4)

U and V are the left and right singular vectors corresponding to the singular values present along the diagonal in the matrix E.

Eigenvalues and eigenvectors are obtained from either of the relationships mentioned in Eqs 3 or 4. In the proposed research, MATLAB is used for identifying all the eigenvectors and eigenvalues of the covariance matrix. The POD basis, Φ, is formed by arranging all the eigenvectors (ϕ_i)in a matrix. Those eigenvectors correspond to the largest eigenvalues (λ_i) of the covariance matrix.

GALERKIN PROJECTION AND THE REDUCED-ORDER MODEL

Once the POD bases are obtained, they are used for creating the reduced-order model. The reduced-order model can be found by the following methods

- By Galerkin Projection of the original analytical equations onto the POD basis functions
- By Galerkin Projection of the discrete form of the governing equations onto the POD basis functions

By performing the latter type of projection, the actual solution will be now represented by the following relation

$$T(x,t) = \sum_{i=1}^{N} a_i(t)\phi_i(x) \tag{5}$$

where $a_i(t)$ are the POD coefficients and ϕ_i s are the orthonormal POD bases. The reduced-order model shows how the POD coefficients evolve. The POD coefficients are found by calculating $a_i(t) = \phi_i^T T(x,t)$.

BOUNDARY CONDITION INDEPENDENT NATURE OF REDUCED-ORDER MODEL

The important contribution of the current work is to show that a boundary condition independent reduced-order model can be obtained using the POD Methodology. The superposition principle of the heat equation allows us to achieve this goal. The principle of superposition is used in the construction of the snapshot matrix. Since the transient equation captures a majority of the characteristics of the detailed geometry, transient solutions of select boundary conditions are used to construct the snapshot matrix. Whether transient solutions are really necessary as opposed to steady-state solutions of different boundary conditions is yet to be investigated.

FINITE VOUME METHOD FOR 1D HEAT EQUATION AND ORDER REDUCTION

The following problem, illustrated by Astrid [11], is used to demonstrate that a boundary condition independent reduced-order model can be generated by the POD-Galerkin approach. The governing heat equation is

$$\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} \tag{6}$$

Length of the 1D rode is 0.1 m. The rod is divided into 400 grid cells. Thermal conductivity is k = 100 $Wm^{-1}K^{-1}$, ρc_p = 10 x 10^6 J/m^3K. Heat conduction is simulated for a time of 240 seconds by discretizing the time domain into 120 time steps. The discretized form of the equations based on the fully implicit method as described in Versteeg and Malasekara [14].

By re-arranging the coefficients of the temperature variable, and representing the resultant arrangement in a standard matrix form, the following is obtained:

$$AT(t+\Delta t) = A^0 T(t) + B\,u(t) \tag{7}$$

In order to project this model into the POD subspace, we replace $\mathrm{T(t)} = \Phi \mathrm{a(t)}$. The Galerkin projection of the discreet form of the equations on to the POD basis Φ then yields

$$\Phi^T A \Phi a(t+\Delta t) = \Phi^T A^0 \Phi a(t) + \Phi^T B\,u(t) \tag{8}$$

Equation 8 is the reduced-order model of the 1D heat equation discretized by the finite volume method. The same methodology can be easily extended to 2D and 3D geometries. This methodology can be implemented in CFD codes through macros or user defined functions. As opposed to the reduce-then-interconnect method [6][7], this is an interconnect-then-reduce method. The reason for using FVM is that major CFD software packages that are being used in the electronics cooling industry (Flotherm, Icepak e.t.c.,) are FVM-based codes. Once the methodology for generating reduced-order models for complex 3D geometries is established, it can be easily integrated to such codes.

1D HEAT EQUATION WITH DIFFERENT BOUNDARY CONDITIONS

The linear transient 1D heat equation is a widely popular example for demonstrating the concept of reduced-order model generation using POD due to the simplicity of the equation and the availability of analytical solutions to validate the results. In the present study, the emphasis is placed on generating a reduced-order model that is valid for any boundary condition.

A heat source is attached to a single cell in the centre of the rod. The boundary conditions on the left and right side are varied. The goal is to identify a single set of POD basis needed to create a reduced-order model for the 1D rod with heat generation that is completely boundary condition independent. The initial condition is 100 deg C and is kept constant. Effects of initial conditions are to be investigated later. There are three cases from which the POD basis is constructed. The fourth case is used for validation purposes. It was observed that POD basis derived from convective boundary conditions behaves the same way as fixed temperature boundary conditions.

Case 1: Right Side – Fixed Temperature and Left Side - Insulated

POD basis was obtained from the snapshot of simulations with two fixed temperature values (0 deg C and 80 deg C) on the right side, while the left side was insulated. Figure 1 shows the application of the obtained POD basis to simulate a right side temperature of 25 deg C while the left side remains insulated. When the problem was solved with POD basis from a single temperature boundary condition (solution of either 0 deg C or 80 deg C), the obtained solution was not accurate. It was observed that snapshots from both cases (0 deg C and 80 deg C) boundary conditions are necessary to construct the POD basis. Figure 2 shows the application of the same POD basis to simulate a condition with right side at a temperature of 60 deg C, while the left side remains insulated.

Case 2: Both sides insulated

Figure 3 shows that POD with 6 basis vectors accurately replicated the behavior when both sides are insulated.

Case 3: Left side – Fixed Temperature; Right side – Insulated

The first case is repeated with the left side at a fixed temperature. Similar to the Case 1, two extreme boundary conditions are simulated (0 deg C and 80 deg C). POD basis constructed from the solution of the two extreme boundary conditions is used for simulating a boundary condition that lies within the extremes (25 deg C). This is shown in Fig. 4.

Case 4: Both sides at fixed temperature

This case serves as the validation case, while cases 1, 2 and 3 serves as the reduced-order model generation cases. For Case 4, a POD basis is generated from the solutions of Cases 1, 2 and 3. The resulting POD basis has 12 vectors. The reduced order model obtained with this POD basis is used for simulating the problem with both ends at a fixed temperature of 0 deg C. The resulting POD basis will now serve as the reduced-order model. The results for this case are compared in Fig. 5.

CONCLUSION

Thus, a boundary condition independent reduced-order model is constructed for a rod with a single heat source in its center with asymmetric boundary conditions on its ends. Error in all cases is less than 1%. Also, an order reduction from computation of 400 variables has been reduced to 12 variables. A total of five snapshots (2 from fixed temperature boundary condition on each end and 1 from both sides being insulated) are sufficient to generate a reduced order model of the rod that is boundary condition independent. With the final set of 12 POD bases, the solution to the problem can be predicted for any boundary condition. An extension of this to 3D case would imply that with 13 snapshots (2D case would require 9 snapshots), one should be able to obtain a boundary condition independent reduced order model.

FIGURES

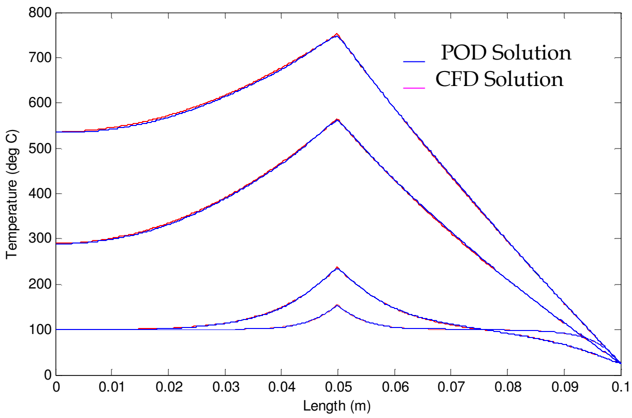

Fig.1 POD and CFD solution for a right side temperature of 25 deg C

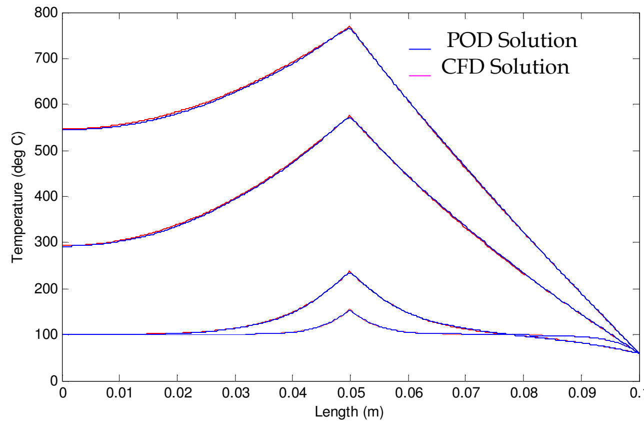

Fig. 2 POD and CFD solution for a right side temperature of 60 deg C

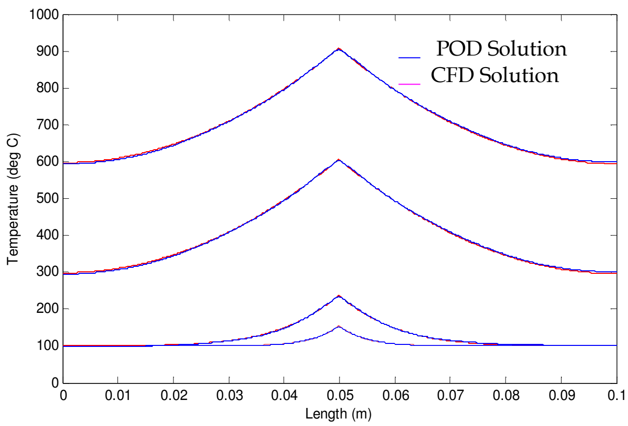

Fig. 3 POD and CFD solution for both sides insulated

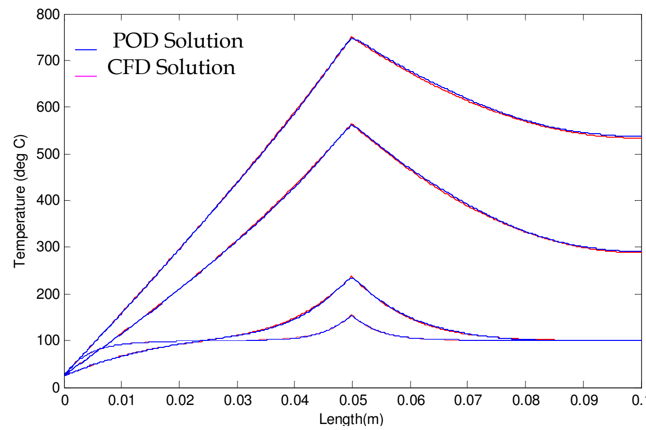

Fig. 4 POD and CFD solution for a left side temperature of 25 deg C

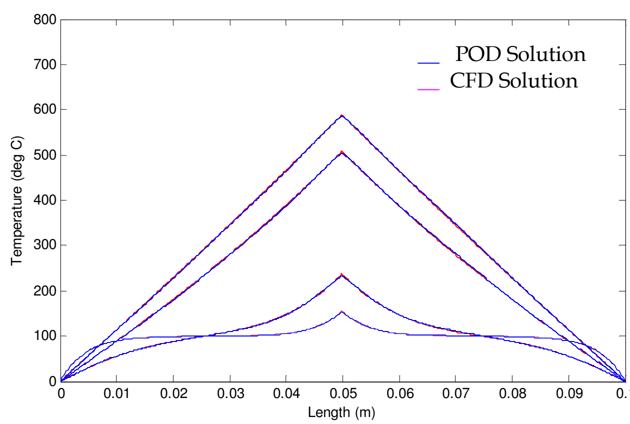

Fig. 5 POD and CFD solution for both ends at fixed temperature. (The results are obtained from POD basis generated based on snapshot of temperatures from cases 1, 2 and 3.)

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